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21MATCS41

Fourth Semester B.E./B.Tech. Degree Examination, June/July 2025

Mathematical Foundations for Computing, Probability and Statistics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define tautology. Show that .
$\neg(p \to q) \to \neg q$ is a tautology by constructing truth table. **(06 Marks)**

b. Suppose the Universe consists of all integers. Consider the following open statements :
$P(x) : x \le 3$, $q(x) = x+1$ is odd, $r(x) : x > 0$
Write down the truth values of the following statements :
(i) $q(1)$
(ii) $\neg p(3)$
(iii) $p(3) \wedge q(4)$
(iv) $p(7) \vee q(7)$
(v) $\neg p(3) \vee r(0)$
(vi) $p(0) \to q(0)$
(vii) $\neg[p(-4) \vee q(-3)]$ **(07 Marks)**

c. Give an indirect proof and proof by contradiction for the statement "If n^2 is an odd then n is odd." **(07 Marks)**

OR

2 a. Prove the following using laws of logic,
$[\neg p(\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. **(07 Marks)**

b. Test the validity of the following argument using Rules of Inference
If Ravi goes out with friends, he will not study
If Ravi does not study, his father becomes angry
His father is not angry
$\therefore$ Ravi has not gone out with friends **(07 Marks)**

c. Define (i) Open statements (ii) Quantifiers **(06 Marks)**

Module-2

3 a. Consider the function $f : R \to R$ defined by $f(x) = 2x + 5$. Let a function $g : R \to R$ be defined by $g(x) = \frac{1}{2}(x-5)$. Prove that g is an inverse of f. **(06 Marks)**

b. Relation R defined on the set A = {1, 2, 3, 4} is shown in the following matrix.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Write down R. Verify that R is an equivalence relation and determine the partition induced. **(07 Marks)**

c. In the following figure, determine (i) a path from b to d (ii) a walk from b to d that is not a trail (iii) a closed walk from b to b that is not a circuit (iv) a circuit from b to b that is not a cycle. **(07 Marks)**

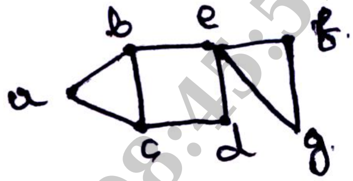

Fig. Q3 (c)

OR

4 a. Let $f : A \to B$, $g : B \to C$, are any two functions. Prove that,

(i) If $g \circ f : A \to C$ is onto, then g is onto.

(ii) If $g \circ f : A \to C$ is one-to-one, then f is one-to-one. **(06 Marks)**

b. Let $A = \{1,2,3,4,6,12\}$. Define a relation R on A by aRb if and only if "b is a multiple of a". Write down the relation R. Prove that R is a Poset and draw the Hasse diagram for this relation. **(07 Marks)**

c. Define graph isomorphism. Determine whether the following graphs are isomorphic or not. **(07 Marks)**

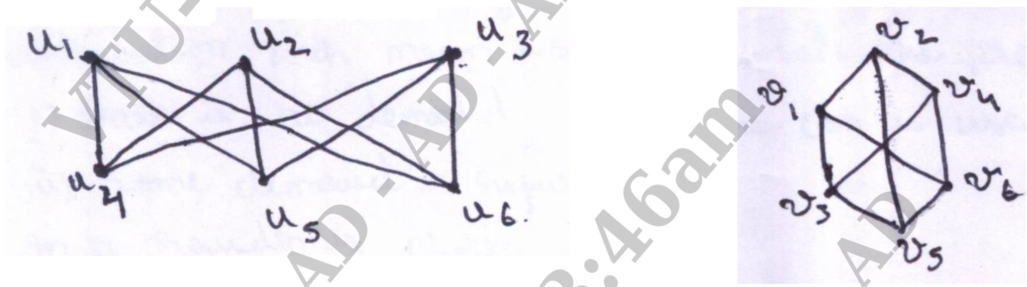

Fig. Q4 (c)

Module-3

5 a. If P is the pull required to lift a load W by means of a pulley block, find a law in the form of a straight line $P = mW + C$ by using the following data. Compute P when W = 150 kg wt.

P	12	15	21	25
W	50	70	100	120

(06 Marks)

b. Find the correlation co-efficient between x and y :

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

(07 Marks)

c. The regression equations of z variables x and y are $x = 0.7y + 5.2$ and $y = 0.3x + 2.8$. Find the means $\bar{x}$ and $\bar{y}$. Also find the coefficient of correlation between x and y. **(07 Marks)**

OR

6 a. Following table gives the data on rainfall and discharge in a certain river. Obtain the line of regression of y on x.

Rainfall –x (inch)	1.53	1.78	2.60	2.95	3.42
Discharge y (1000 cc)	33.5	36.3	40.0	45.8	53.5

What is the estimated discharge corresponding to rainfall of 3 inch. **(06 Marks)**

b. Find the rank correlation for the following data :

x	78	36	98	25	75	82	90	62	65	39
y	84	51	91	60	68	62	86	58	53	47

(07 Marks)

c. Fit a parabola $y = a + bx + cx^2$ to the following data :

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(07 Marks)

Module-4

7 a. A random variable X has the following probability function:

x	1	2	3	4	5	6
p(x)	0.2	K	0.1	2K	0.3	3K

Find K and calculate mean and standard deviation of X. **(06 Marks)**

b. A car hire firm has 2 cars which it hires out day by day. The number of demand for a car is distributed as a Poisson distribution with mean 1.5. Calculate the probability that
(i) There is no demand
(ii) One car is used.
(iii) Some demand is refused.
On a randomly choosen day. **(07 Marks)**

c. In a normal distribution, 7% are under 35 and 11% are over 63. Find the mean and standard deviation. Given A(1.48) = 0.43 and A(1.23) = 0.39 in the usual notation. **(07 Marks)**

OR

8 a. A function is defined as, $P(x) = \begin{cases} \frac{1}{18}(2x+3), & \text{if } 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$. Show that P(x) is a density function. Find mean of x. **(06 Marks)**

b. The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If 6 bombs are dropped, find the probability that, (i) Exactly 2 will strike the target (ii) At least 1 will strike the target. **(07 Marks)**

c. The life of a certain type of electrical lamp is normally distributed with a mean of 2040 hrs and standard deviation of 60 hrs. In a consignment of 2000 lamps, find how many would be expected to burn for, (i) more than 2150 hrs (ii) less than 1950 hrs (iii) between 1920 hrs and 2160 hrs. Given A(1.83) = 0.4664, A(1.5) = 0.4332, A(2) = 0.4772. **(07 Marks)**

Module-5

9 a. Explain the terms : (i) Sample (ii) Null hypothesis (iii) Type I and Type II error **(06 Marks)**

b. A sample height of 6400 soldiers has a mean of 172.34 cms and a standard deviation of 6.5 cms, while a sample of height of 1600 soldiers has a mean of 174.12 cms and a standard deviation of 6.4 cms. Does the data indicate that sailors are on the average taller than soldiers? Use the left tailed test at 0.01 level of significance. Given $-Z_C = -2.33$ **(07 Marks)**

c. The following table gives the number of road accidents that occurred in a city during the various days of a week. Test the hypothesis that the accidents are uniformly distributed over all the days of the week. Given $\chi^2_{0.01}(6) = 16.81$

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents	14	16	8	12	11	9	14

(07 Marks)

OR

10 a. Find the joint distributions of X and Y, which are independent random variables with the following respective distributions and find mean X, Y and XY. **(06 Marks)**

X	1	2
P(X)	0.7	0.3

Y	-2	5	8
P(Y)	0.3	0.5	0.2

b. A die was thrown 1200 times and the number 6 was obtained 236 times. Can the die be considered fair at 0.01 level of significance? ($Z_C = 2.58$). **(07 Marks)**

c. A random sample of 10 students have the following I.Q.
70, 120, 110, 101, 88, 83, 95, 98, 107, 100
Does this data support the hypothesis that the population mean I.Q. is 100 at 5% level of significance? Given that $t_{0.05}(g) = 2.26$. **(07 Marks)**

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