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## Second Semester B.E. Degree Examination, June/July 2025 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

*Note: Answer any FIVE full questions, choosing ONE full question from each module.*

### Module-1

- 1 a. Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, -1, 2)$ . (06 Marks)
- b. If  $\vec{F} = \nabla(xy^3z^2)$ , find  $\text{div } \vec{F}$  and  $\text{curl } \vec{F}$  at the point  $(1, -1, 1)$ . (07 Marks)
- c. Show that  $\vec{F} = (y+z)\mathbf{i} + (z+x)\mathbf{j} + (x+y)\mathbf{k}$  is irrotational. Also find a scalar function  $\phi$  such that  $\vec{F} = \nabla\phi$ . (07 Marks)

### OR

- 2 a. If  $\vec{F} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$ , evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is the curve represented by  $x = t, y = t^2, z = t^3, -1 \leq t \leq 1$ . (06 Marks)
- b. Using the Green's theorem, evaluate  $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$ , where  $C$  is the boundary of the region enclosed by  $y = \sqrt{x}$  and  $y = x^2$ . (07 Marks)
- c. If  $\vec{F} = (2x^2 - 32)\mathbf{j} - 2xy\mathbf{j} - 4x\mathbf{k}$ , evaluate  $\iiint_V \nabla \cdot \vec{F} dv$  where  $v$  is the region bounded by the planes  $x = 0, y = 0, z = 0$  and  $2x + 2y + z = 4$ . (07 Marks)

### Module-2

- 3 a. Solve  $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ . (06 Marks)
- b. Solve  $(D^2 + 1)y = \tan x$  by the method of variation of parameter. (07 Marks)
- c. Solve  $x^2y'' - 3xy' + 5y = 3 \sin(\log x)$ . (07 Marks)

### OR

- 4 a. Solve  $(D^2 + 4)y = 2^{-x} + \cos 2x$ . (06 Marks)
- b. Solve  $(2x + 1)^2 y'' - 6(2x + 1)y' + 16y = 8(2x + 1)^2$ . (07 Marks)
- c. The differential equation of a simple pendulum  $\frac{d^2x}{dt^2} + w^2x = F \sin nt$ , where  $w$  and  $F$  are constants. If at  $t = 0, x = 0$  and  $\frac{dx}{dt} = 0$ , determine the motion when  $n = w$ . (07 Marks)

### Module-3

- 5 a. Form the partial differential equation by eliminating the arbitrary function from  $lx + my + nz = \phi(x^2 + y^2 + z^2)$ . (06 Marks)
- b. Solve  $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x + 3y)$ . (07 Marks)
- c. Derive one-dimensional wave equation in usual notations. (07 Marks)

OR

- 6 a. Solve  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z$  (06 Marks)
- b. Solve  $\frac{\partial^2 z}{\partial y^2} = z$  given that  $y = 0, z = e^z$  and  $\frac{\partial z}{\partial y} = e^{-x}$  (07 Marks)
- c. Find the various possible solution of the one dimensional heat equation  $u_t = c^2 u_{xx}$  by the method of separation of variable. (07 Marks)

**Module-4**

- 7 a. Discuss the nature of the series  

$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}} x^n$$
 (06 Marks)
- b. With usual notation prove that  

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$$
 (07 Marks)
- c. Express  $x^3 - 5x^2 + x + 2$  in terms of Legendre's polynomials. (07 Marks)

OR

- 8 a. Discuss the nature of the series  

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots \quad (x > 0)$$
 (06 Marks)
- b. Prove the orthogonality property of Bessel's function as  $\int_0^1 x J_n(\alpha x) J_n(\beta x) dx = 0, \alpha \neq \beta$ . (07 Marks)
- c. If  $x^3 + 2x^2 - 4x + 5 = a p_3(x) + b p_2(x) + c p_1(x) + d p_0(x)$ , find a, b, c and d. (07 Marks)

**Module-5**

- 9 a. Using Newton's forward difference formula find  $f(1.4)$

x:	1	2	3	4	5
f(x):	10	26	58	112	194

- (06 Marks)
- b. Find the real root of  $x e^x - \cos x = 0$  correct to three decimal places lying in the interval  $(-5, -6)$  using Regula Falsi method. (07 Marks)
- c. Evaluate  $\int_0^1 \frac{x}{1+x^2} dx$  by using Simpson's  $\left(\frac{1}{3}\right)^{th}$  rule taking six equal strips. (07 Marks)

OR

- 10 a. Show that a root of the equation  $x^3 + 5x - 11 = 0$  lies between 1 and 2. Find the root by Newton's Raphson method carryout two iterations. (06 Marks)
- b. Find  $f(9)$  from the data by Newton's divided difference formula.

x:	5	7	11	13	17
y:	150	392	1452	2366	5202

- (07 Marks)
- c. Evaluate  $\int_4^{5.2} \log_e x dx$  taking six equal strips by applying Weddle's rule. (07 Marks)

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