CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025
Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. **(06 Marks)**

b. A periodic function of period $\frac{2\pi}{\omega}$ is defined by, $f(t) = \begin{cases} E\sin\omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$ **(07 Marks)**

c. Solve : $y'' - y' - 2y = 0$; given y(0) = 0 and $y'(0) = 6$ by Laplace transformation method. **(07 Marks)**

OR

2 a. Find $L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\}$. **(06 Marks)**

b. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s^2+1)(s^2+9)}\right\}$. **(07 Marks)**

c. Using unit step function, find the Laplaces transform of, $f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ \sin 2t, & \pi \le t \le 2\pi \\ \sin 3t, & t \ge 2\pi \end{cases}$. **(07 Marks)**

Module-2

3 a. Obtain the Fourier Series for the function, $f(x) = x^2$, $0 < x < 2\pi$. **(06 Marks)**

b. Find the Fourier series of f(x),
Where $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. **(07 Marks)**

c. Express f(x) = x as a half-range cosine series in 0 < x < 2. **(07 Marks)**

OR

4 a. Obtain Fourier series for the function, $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$. **(06 Marks)**

b. Find the Fourier half-range cosine series of the function $f(x) = (x+1)$, in (0, 1). **(06 Marks)**

c. Compute the first harmonic of the Fourier series of f(x) given in the following table :

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(08 Marks)

Module-3

5 a. Find the z-transform of, $3n - 4\sin\left(\frac{n\pi}{4}\right) + 5a$. **(06 Marks)**

b. Compute the inverse $z -$ transform of, $\frac{2z^2 + 3z}{(z+2)(z-4)}$. **(07 Marks)**

c. Find the Fourier transform of, $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$, Hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. **(07 Marks)**

OR

6 a. Find the Fourier sine transform of e^{-ax}. **(06 Marks)**

b. If $U(z) = \frac{2z^2 + 5z + 14}{(z-1)^4}$, evaluate u_2 and u_3. **(07 Marks)**

c. Using the z-transform, solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, $u_0 = 0$, $u_1 = 1$. **(07 Marks)**

Module-4

7 a. Find by Taylor's series method the value of y at x = 0.1 and x = 0.2 to four places of decimals from,
$\frac{dy}{dx} = x^2y - 1$, y(0) = 1. **(06 Marks)**

b. Apply Runge-Kutta fourth order method to find an approximate value of y when x = 0.2, given $\frac{dy}{dx} = x + y$, y(0) = 1. **(07 Marks)**

c. If $\frac{dy}{dx} = 2e^x - y$, y(0) = 2, y(0.1) = 2.010, y(0.2) = 2.04 and y(0.3) = 2.09, find y(0.4) by employing the Milne's predictor-corrector formula, use corrector formula twice. **(07 Marks)**

OR

8 a. Using modified Euler's method, solve the IVP $\frac{dy}{dx} = x + \sqrt{y}$, y(0) = 1 at x = 0.2, perform three modifications. **(06 Marks)**

b. Using the fourth order Runge-Kutta method, solve the IVP $\frac{dy}{dx} = \frac{1}{x+y}$ at the point x = 0.5. Given that $y(0.4) = 1$. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$, y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Determine y(1.4) by Adams-Bashforth method. **(07 Marks)**

Module-5

9 a. Using Runge-Kutta method of fourth order solve the differential equation, $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$, with y(0) = 1, $y'(0) = 0$ at x = 0.2. **(06 Marks)**

b. Derive Euler's equation in the standard form, $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

c. On which curve the functional,

$\int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 2xy\right)dy$, $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$ be extremized. **(07 Marks)**

OR

10 a. Apply Milne's method to compute y(0.8), given that $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ and

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

(06 Marks)

b. Prove that the geodesics on a plane are straight line. **(07 Marks)**

c. Find the extremal of the functional, $I = \int_0^{\frac{\pi}{2}} \left(y'^2 - y^2 + 4y\cos x\right)dx$, given that $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 0$. **(07 Marks)**

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