

CBCS SCHEME

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18MATDIP31

Third Semester B.E. Degree Examination, Dec.2024/Jan.2025 Additional Mathematics – I

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. If $2\cos\theta = x + \frac{1}{x}$, show that $2\cos n\theta = x^n + \frac{1}{x^n}$ and $2i\sin n\theta = x^n - \frac{1}{x^n}$. Also show that $\frac{x^{2n}-1}{x^{2n}+1} = i \tan n\theta$. (08 Marks)
- b. Find the real part of $\frac{1}{1+\cos\theta+\sin\theta}$. (06 Marks)
- c. Express $\left(\frac{3+4i}{3-4i}\right)$ in $a+ib$ form. (06 Marks)

OR

- 2 a. If $\vec{a} = 2i + 3j - 4k$ and $\vec{b} = 8i - 4j - k$ prove that $\vec{a}$ is perpendicular to $\vec{b}$. Also find $|\vec{a} \times \vec{b}|$. (08 Marks)
- b. If $\vec{a} = 3i - 2j - 4k$ and $\vec{b} = i + j - 2k$, find:
i) $|2\vec{a} + 3\vec{b}|$ ii) $|\vec{a} \cdot \vec{b}|$ iii) angle between $\vec{a}$ and $\vec{b}$. (06 Marks)
- c. Find a unit vector normal to both the vectors $4i - j + 3k$ and $-2i + j - 2k$. Find also the sine of the angle between them. (06 Marks)

Module-2

- 3 a. Obtain the Maclaurin's series expansion of $\sqrt{1+\sin 2x}$ up to the form containing x^4 . (08 Marks)
- b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$. (06 Marks)
- c. If $u = f(x-y, y-z, z-x)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. (06 Marks)

OR

- 4 a. Show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ by using Maclaurin's series notation. (08 Marks)
- b. If $u = e^{ax+by} f(ax-by)$, prove that $b \frac{\partial u}{\partial x} + a \frac{\partial u}{\partial y} = 2abu$ (06 Marks)
- c. If $u = x + y$ and $v = \frac{y}{x+y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$. (06 Marks)

Module-3

- 5 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where 't' is the time. Find the velocity and acceleration at $t = 1$ in the direction $i - 3j + 2k$. (08 Marks)
- b. Find the unit vector normal to the surface $x^2 - y^2 + z = 2$ at the point $(1, -1, 2)$. (06 Marks)
- c. Prove that $\vec{d} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is irrotational. (06 Marks)

OR

- 6 a. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$, where $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$. (08 Marks)
- b. If $\vec{A} = xyi + y^2zj + z^2yk$, find $\text{curl}(\text{curl } \vec{A})$. (06 Marks)
- c. Find 'a' if the vector field $\vec{F} = (ax + 3y + 4z)i + (x - 2y + 3z)j + (3x + 2y - z)k$ is Solenoidal. (06 Marks)

Module-4

- 7 a. Obtain the reduction formula for $\int_0^{\pi/2} \sin^n x dx$ ($n > 0$). (08 Marks)
- b. Evaluate $\int_0^{\infty} \frac{x^4}{(1+x^2)^4} dx$. (06 Marks)
- c. Evaluate: $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$. (06 Marks)

OR

- 8 a. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x dx$, where n is a positive integer. (08 Marks)
- b. Evaluate $\int_0^{\pi/2} \sin^3 x \cos^7 x dx$. (06 Marks)
- c. Evaluate $\int_0^1 \int_0^1 \int_0^1 xyz dx dy dz$. (06 Marks)

Module-5

- 9 a. Solve $[(\cos x \cdot \tan y + \cos(x+y))]dx + [\sin x \sec^2 y + \cos(x+y)]dy = 0$ (08 Marks)
- b. Solve $\frac{dy}{dx} + y \cot x = \cos x$. (06 Marks)
- c. Solve $(x^2 + y)dx + (y^3 + x)dy = 0$. (06 Marks)

OR

- 10 a. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$ (08 Marks)
- b. Solve $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$. (06 Marks)
- c. Solve $x \frac{dy}{dx} + y = x^3 y^6$. (06 Marks)