USN | | | | | | | | | | **MATDIP401**

Fourth Semester B.E. Degree Examination, June/July 2013

Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If ℓ, m, n are the direction cosines of a line then prove that $\ell^2 + m^2 + n^2 = 1$. **(06 Marks)**

b. Show that the direction ratios of three lines 2, 1, 1; $4, \sqrt{3}-1, -\sqrt{3}-1$ and $4, -\sqrt{3}-1, \sqrt{3}-1$ are equally inclined to one-another. **(07 Marks)**

c. Find the expression for the angle between two lines whose direction cosines are ℓ_1, m_1, n_1 and ℓ_2, m_2, n_2. **(07 Marks)**

2 a. Find the equation of the plane passing through three points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3). **(06 Marks)**

b. Find the equation of the plane through the point (3, –3, 1) and its normal to the line joining the prints (3, 2, –1) and (2, –1, 5). **(07 Marks)**

c. Find the equation of the plane through (1, –2, 2), (–3, 1, –2) and perpendicular to the plane $2x - y - z + 6 = 0$. **(07 Marks)**

3 a. If $\vec{a} = r\cos\theta\sin\phi\hat{i} + r\sin\theta\sin\phi\hat{j} + r\cos\theta\hat{k}$, then show that $|\vec{a}| = r$. **(06 Marks)**

b. Prove that $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. **(07 Marks)**

c. Show that the position vectors of the vertices of a triangle $\vec{a} = 3(\sqrt{3}\hat{i} - \hat{j})$, $\vec{b} = 6\hat{j}$, $\vec{c} = 3(\sqrt{3}\hat{i} + \hat{j})$ form an isosales triangle. **(07 Marks)**

4 a. Find the unit tangent vector to the space curve $x = \cos t^2$, $y = \sin t^2$ and $z = 0$. **(06 Marks)**

b. Prove that $\frac{d}{dt}[\vec{F},\vec{G},\vec{H}] = \left[\frac{d\vec{F}}{dt},\vec{G},\vec{H}\right] + \left[\vec{F},\frac{d\vec{G}}{dt},\vec{H}\right] + \left[\vec{F},\vec{G},\frac{d\vec{H}}{dt}\right]$ **(07 Marks)**

c. Find the tangent and normal components of its acceleration at t = 1 of a particle moves along the curve $\vec{r} = t^2\hat{i} - t^3\hat{j} + t^4\hat{k}$. **(07 Marks)**

5 a. Prove that $\text{div}(\vec{A} + \vec{B}) = \text{div}\vec{A} + \text{div}\vec{B}$. **(06 Marks)**

b. If $\vec{A}$ is a vector function and ϕ is a scalar function then $\nabla\times(\phi\vec{A}) = \phi(\nabla\times\vec{A}) + \nabla\phi\times\vec{A}$. **(07 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at (2, –1, 2). **(07 Marks)**

6 a. Find L[f(t)] given that $f(t) = \begin{cases} 0 & \text{for} \quad 0 < t < 2 \\ 4 & \text{for} \quad t > 2 \end{cases}$. **(05 Marks)**

b. Find: i) $L[\cos^2 4t]$; ii) $L[\sin 2t \cos 3t]$; iii) $L\left[\frac{1-\cos t}{t}\right]$ **(15 Marks)**

7 a. If L[f(t)] = F(s), show that $L\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$. **(05 Marks)**

b. Find: i) $L^{-1}\left[\frac{5s+1}{s^2+16}\right]$

ii) $L^{-1}\left[\frac{1}{(s+1)(s+2)(s+3)}\right]$

iii) $L^{-1}\left[\frac{s}{(s+2)^3}\right]$ **(15 Marks)**

8 a. Using Laplace transform solve:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-t}, \qquad y(0) = 0 = y'(0).$$ **(10 Marks)**

b. Solve the system of equation using Lapalce transforms $\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$. **(10 Marks)**

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