CBCS SCHEME

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17MAT11

First Semester B.E. Degree Examination, June/July 2019
Engineering Mathematics – I

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the n^{th} derivative of $\sin 2x \ \sin 3x$. **(06 Marks)**

b. Find the angle between the two curves $r = \dfrac{a}{1+\cos\theta}$ and $r = \dfrac{b}{1-\cos\theta}$ **(07 Marks)**

c. Find the radius of curvature for the curve $x^3 + y^3 = 3xy$ at (3/2, 3/2). **(07 Marks)**

OR

2 a. If $y = \cos(m \log x)$ then prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2 + n^2)y_n = 0$. **(06 Marks)**

b. With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$. **(07 Marks)**

c. Find the pedal equation of the curve $r^m = a^m \cos m\theta$. **(07 Marks)**

Module-2

3 a. Find the Taylor's series of log(cosx) in powers of $(x - \pi/3)$ upto fourth degrees terms. **(06 Marks)**

b. If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$ by using Euler's theorem. **(07 Marks)**

c. If $u = \dfrac{yz}{x}, v = \dfrac{xz}{y}, w = \dfrac{xy}{z}$ then find $J = \dfrac{\partial(uvw)}{\partial(xyz)}$. **(07 Marks)**

OR

4 a. Evaluate $\lim\limits_{x \to 0}\left(\dfrac{\tan x}{x}\right)^{1/x^2}$. **(06 Marks)**

b. Using Maclaurin's series, prove that $\sqrt{1+\sin 2x} = 1 + x - \dfrac{x^2}{\lfloor 2} - \dfrac{x^3}{\lfloor 3} + \dfrac{x^4}{\lfloor 4} - + - - - -$. **(07 Marks)**

c. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then prove that $\dfrac{1}{2}\dfrac{\partial u}{\partial x} + \dfrac{1}{3}\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\dfrac{\partial u}{\partial z} = 0$. **(07 Marks)**

Module-3

5 a. A particle moves along the cuvre $\vec{r} = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$. Find the components of velocity and acceleration in the direction of $\hat{i} - 3\hat{j} + 2\hat{k}$ at $t = 0$. **(06 Marks)**

b. Find the constant a and b such that $\vec{F} = (axy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (bxz^2 - y)\hat{k}$ is irrotational and find scalar potential function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

c. Prove that $\text{curl}(\phi\vec{A}) = \phi\,\text{curl}\,\vec{A} + \text{grad}\,\phi \times \vec{A}$. **(07 Marks)**

OR

6 a. Show that vector field $F = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2}$ is both solenoidal and irrotational. **(06 Marks)**

b. If $\vec{F} = (x+y+1)\vec{i} + \vec{j} - (x+y)\vec{k}$ then prove that $\vec{F} = \text{curl}\,\vec{F} = 0$. **(07 Marks)**

c. Show that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(07 Marks)**

Module-4

7 a. Obtain reduction formula for $\int \sin^n x\, dx (n > 0)$. **(06 Marks)**

b. Solve the differential equation $\dfrac{dy}{dx} + y\cot x = \cos x$. **(07 Marks)**

c. Find the orthogonal trajectory of the curve $r = a(1+\sin\theta)$. **(07 Marks)**

OR

8 a. Evaluate $\int_0^{\pi/2} \sin^7\theta \cos^6\theta\, d\theta$. **(06 Marks)**

b. Solve the differential equation : $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$. **(07 Marks)**

c. If the temperature of air is 30°C and the substance cools from 100°C to 70°C in 15 mins. Find when the temperature will be 40°C. **(07 Marks)**

Module-5

9 a. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \\ 2 & -2 & 3 & 7 \end{bmatrix}$ by reducing to Echelon form. **(06 Marks)**

b. Find the largest eigen value and egien vector of the matrix : $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking initial vector as $[1\ 1\ 1]^T$ by using Rayleigh's power method. Carry out five iteration. **(07 Marks)**

c. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form, using orthogonal transformation. **(07 Marks)**

OR

10 a. Solve the system of equations

$$10x + y + z = 12$$
$$x + 10y + z = 12$$
$$x + y + 10z = 12$$

by using Gauss-Seidel method. Carry out three iterations. **(06 Marks)**

b. Diagonalise the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. **(07 Marks)**

c. Show that the transformation

$y_1 = x_1 + 2x_2 + 5x_3$
$y_2 = 2x_1 + 4x_2 + 11x_3$
$y_3 = -x_2 + 2x_3$

is regular. Write down inverse transformation. **(07 Marks)**