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10MAT21

Second Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The solution of the first order equation, $P^2 - 5P - 14 = 0$ is,
A) $(y-7x-c)(y-2x-c)=0$ B) $(y-7x-c)=0$
C) $(x+y-c)=0$ D) $(y-2x-c)=0$

ii) The solution of the equation $P = \log(px - y)$ is,
A) $y = cx + \sin^{-1} c$ B) $y = cx - e^c$ C) $y = cx + e^c$ D) $y = cx - \sin^{-1} c$

iii) If $\frac{dx}{dp} + \frac{2}{P}x = 2$ be the linear differential equation then the integrating factor is,
A) $\frac{1}{p^2}$ B) p^2 C) $-p^2$ D) logp

iv) The differential equation of atmospheric pressure at any height Z is,
A) $\frac{dP}{dZ} = \frac{-\rho g}{K}$ B) $\rho = \frac{P}{K}$ C) $P = \frac{\rho}{K}$ D) $P = \rho K$

b. Solve : $xy\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}(3x^2 - 2y^2) = 6xy$ **(04 Marks)**

c. Solve : $y = 2Px + P^m$ **(06 Marks)**

d. Solve : $e^{3x}(P-1) + P^3 e^{3y} = 0$ **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) A general solution of a linear differential equation of 3rd order contains,
A) One constant B) Two constants C) Three-constants D) n-constants

ii) The solution of the differential equation, $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$ is,
A) $y = C_1e^{-x} + C_2e^{2x}$ B) $y = C_1 + C_2e^{2x}$ C) $y = (C_1 + C_2x)e^{2x}$ D) $C_1x = C_2$

iii) Particular integral of the differential equation $\frac{d^2y}{dx^2} + y = \cos 2x$ is,
A) $\frac{1}{5}\sin 2x$ B) $-\frac{1}{3}\cos 2x$ C) $\frac{1}{3}\cos 2x$ D) $\frac{1}{3}\sin 2x$

iv) Particular integral of the differential equation, $y'' + 2y = 2$ is,
A) 1 B) 2 C) 3 D) None of these

b. Solve : $e^x\frac{d^2y}{dx^2} + 2e^x\frac{dy}{dx} + e^x y = x^2$ **(04 Marks)**

c. Solve the initial value problem, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y + 2\cosh x = 0$, given $y = 0$, $\frac{dy}{dx} = 1$ at $x = 0$. **(06 Marks)**

d. Solve : $\frac{dx}{dt} + 4x + 3y = t$; $\frac{dy}{dt} + 2x + 5y = e^t$ **(06 Marks)**

3 a. Choose the correct answers for the following : (04 Marks)

i) The Wronskian of $u = xe^x$ and $v = e^x$ is,

A) xe^x B) x^2e^x C) $-e^{2x}$ D) $xe^x + e^x$

ii) A linear differential equation in which there are 2 or more dependent variables and single independent variable is known as,

A) Exact equation B) Bernoulli's equation
C) Simultaneous equation D) Cauchy's linear equation

iii) Cauchy's differential equation is a special case of Legendre's linear equation is,

A) $a = 1 = b$ B) $a = 0,\ b \neq 1$ C) $a = 1, b = 0$ D) $a = 0 = b$

iv) In Frobenius method, equating to zero the coefficient of the lower degree in x gives a quadratic equation known as,

A) Frobenius equation B) Indicial equation
C) Linear equation D) Multiple equation

b. Solve by the method of variation parameter, $y'' + 2y' + 2y = e^{-x}\sec^3 x$ (05 Marks)

c. Solve : $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} + y = x^2$ (05 Marks)

d. Obtain the series solution of the equation, $9x(1-x)\dfrac{d^2y}{dx^2} - 12\dfrac{dy}{dx} + 4y = 0$ (06 Marks)

4 a. Choose the correct answers for the following : (04 Marks)

i) The partial differential equation of $z = f(x^2 + y^2)$ is,

A) $yp - xq = 0$ B) $p^2 + q^2 = 0$ C) $z = p^2q^2$ D) None of these

ii) The solution of the equation $\dfrac{\partial^2 z}{\partial x^2} = xy$ is,

A) $\dfrac{x^3y^3}{6}$ B) $\dfrac{y^3x}{6}$ C) $\dfrac{x^3y}{6} + xf(y) + \phi(y)$ D) $x^2 + y^2 + y$

iii) The solution of partial differential equation $\sqrt{p} + \sqrt{q} = 2$ is,

A) $ax + (2-\sqrt{a})^2 y + c$ B) $\dfrac{y}{x}$ C) $\dfrac{ay}{bx}$ D) $ay - (2-\sqrt{a})^2 x + c$

iv) The partial differential equation of the form $z = px + qy + f(pq)$ in two variables is known as,

A) Clairaut's form B) Lagrange's form C) Bessel's form D) Legrendre's form

b. Solve : $\phi\left(x + y + z, x^2 + y^2 + z^2\right) = 0$ (04 Marks)

c. Solve : $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (06 Marks)

d. Solve by the method of separation of variables. $4\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = 3u$ (06 Marks)

PART – B

5 a. Choose the correct answers for the following : (04 Marks)

i) While computing the volume for $f(x, y, z)$, which of the following expression is true

A) $\iiint_R dxdydz = \int_v dv$ B) $\iiint_R drd\theta dz = \int_v dv$

C) $\iiint_R r\sin\theta . drd\theta d\phi = \int_v dv$ D) None of these

ii) The value of $\int_0^1\int_0^1 xydxdy$ is,

A) 0 B) $\dfrac{1}{4}$ C) $\dfrac{1}{2}$ D) 1

iii) The value of $\left\lceil\frac{1}{4}\right.\left\lceil\frac{3}{4}\right.$ is,

A) $\sqrt{\pi}$ B) $\sqrt{2}\pi$ C) $2\sqrt{\pi}$ D) 2π

iv) The function $\beta(m, n)$ is not equal is,

A) $\frac{\Gamma n.\Gamma m}{\Gamma(n+m)}$ B) $B\frac{(m+1, n)}{m}\times(m+n)$

C) $\frac{B(m, m+1)}{n}$ D) B(n, m)

b. Evaluate $\iint_A xy\,dx\,dy$ where A is the region bounded by the x-axis ordinate x = 2a and the parabola $x^2 = 4ay$. **(06 Marks)**

c. Evaluate $\iiint_R (x+y+z)dx\,dy\,dz$ where R is the tetrahedron, x = 0, y = 0, z = 0 and $x+y+z=1$. **(06 Marks)**

d. Prove that $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$. **(04 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) For any closed surface S the value of $\iint_S \text{Curl}\,\vec{F}.\vec{N}ds =$ ________

A) 1 B) 0 C) $\frac{2}{3}$ D) $\frac{1}{3}$

ii) Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular if,

A) $\vec{a}.\vec{b}=1$ B) $\vec{a}\times\vec{b}=0$

C) $\cos\theta=0$, where θ is angle between $\vec{a}$ and $\vec{b}$ D) None of these

iii) Vector $\vec{F}$ is irrotational if,

A) $\text{Curl}\,\vec{F}=0$ B) $\text{Div}\,\vec{F}=0$ C) $\text{Curldiv}\,\vec{F}=0$ D) None of these

iv) If $\vec{F}$ is a continuously differential vector function in the region E bounded by the closed surface S then Gauss divergence theorem is,

A) $\int_S \vec{F}.\vec{N}ds = \int_E \text{div}\,\vec{F}dV$ B) $\int_S \vec{F}.\vec{N}ds = \int_E \text{Curl}\,\vec{F}dV$

C) $\int_S \vec{F}.\vec{N}ds = \int_E \nabla\phi dV$ D) $\int_S \vec{F}.\vec{N}ds = \int_E \nabla.\nabla\vec{F}dV$

b. If $\vec{F}=3xyi-y^2j$ evaluate $\int_C \vec{F}.d\vec{R}$ where C is the arc of the parabola $y=2x^2$ from (0, 0) to (1, 2). **(04 Marks)**

c. Verify Green's Theorem for $\int_C\left[(xy+y^2)dx+x^2dy\right]$ where C is bounded by y = x and $y = x^2$. **(06 Marks)**

d. Evaluate $\oint_C \vec{F}.d\vec{R}$ Stoke's theorem where $\vec{F}=y^2i+x^2j-(x+z)k$ and C is the boundary of the triangle with vertices at (0, 0, 0), (1, 0, 0) and (1, 1, 0) **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) If $L[f(t)] = F(s)$ then $L[f(at)]$ is equal to

A) $F(sa)$ B) $aF\left(\frac{s}{a}\right)$ C) $\frac{1}{a}F(as)$ D) $\frac{1}{a}F\left(\frac{s}{a}\right)$

ii) $L\left[\frac{1-e^{-at}}{t}\right] =$

A) $\log\left(1+\frac{a}{s}\right)$ B) $\log\left(\frac{a}{s}\right)$ C) $\log\left(1-\frac{a}{s}\right)$ D) $\log\left(\frac{s}{a}\right)$

iii) Laplace transform of $t\sin 2t$ is,

A) $\frac{2}{(s^2+a^2)^2}$ B) $\frac{4s}{(s^2+4)^2}$ C) $\frac{s}{(s^2+4)^2}$ D) $\frac{s}{(s^2-a^2)^2}$

iv) Laplace transform of $e^{at}\delta(t-b)$ is equal to

A) $e^{-(s-a)b}$ B) $e^{(s-a)b}$ C) $e^{-(s+a)b}$ D) None of these

b. Prove that $L[t^n f(t)] = (-1)^n \frac{d^n F(s)}{ds^n}$ **(04 Marks)**

c. Find the Laplace transform of the triangular wave function of period 2a given by,

$$f(t) = \begin{cases} t & 0 < t < a \\ 2a - t & a < t < 2a \end{cases}$$ **(06 Marks)**

d. Express the function in terms of Heaviside's unit step function and find its Laplace transform, $f(t) = \begin{cases} \sin t & 0 \le t \le \pi \\ t & \pi \le t \le 2\pi \\ \cos t & t > 2\pi \end{cases}$. **(06 Marks)**

8 a. Choose the correct answers for the following : **(04 Marks)**

i) Inverse Laplace transform of $\frac{s^3+s^2+6}{s^4}$ is,

A) $1+t+t^2$ B) $2+3t+4t^4$ C) $t+t^2+3t^3$ D) $\frac{1+t^3}{t}$

ii) Inverse Laplace transform of $F(s-a)$ is,

A) $e^{at}L^{-1}[f(t)]$ B) $e^{-at}[f(t)]$ C) $L^{-1}[e^{-at}F(S)]$ D) None of these

iii) The differential equation, $y'' + 2y' + 5y = 8\sin t + 4\cos t$ under the conditions $y(0) = 1$, $y\left(\frac{\pi}{4}\right) = \sqrt{2}$ is an,

A) Initial value problem B) Boundary value problem
C) Both (A) and (B) D) None of these

iv) Inverse Laplace transform of $\frac{F(s)}{s}$ is,

A) $\int_0^\infty f(u)du$ B) $\int_0^t f(u)du$ C) $\int_0^t \frac{F(u)}{u}du$ D) $\int_0^\infty e^{-st}dt$

b. Find the inverse Laplace transform of, $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$. **(04 Marks)**

c. Evaluate using convolution theorem, $L^{-1}\left[\frac{1}{s^2(s^2+16)}\right]$. **(06 Marks)**

d. Solve $y'' + 6y' + 9y = 12t^2e^{-3t}$ given $y(0) = y'(0) = 0$ by Laplace transform method. **(06 Marks)**