machine design & m/c

## 14MAR/MAU/IAE/MDE/MMD/MST/MTH/ MTP/MTE/MTR/MLM/MEA/CAE11

USN

# First Semester M.Tech Degree Examination, Dec.2015/Jan.2016

# Applied Mathematics

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions.***

1 a. What is truncation error? Explain for the series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$ . Compute $e^{0.5}$ by truncating the series to 1, 2, 3, 4 terms. Find truncation error in each step. **(10 Marks)**

b. Write a short note on precision and accuracy. Convert $(0.7)_{10}$ to binary form consisting of 4 and 6 bits. Compute round off error in each case. **(10 Marks)**

2 a. Find a real root of the equation $4e^{-x}\sin x - 1 = 0$. Correct to 3 decimal places by using Newton – Raphson method. Take $x_0 = 0.2$ as initial approximation. **(06 Marks)**

b. Perform 2 iterations of the Muller method. Find the smallest positive root of $x^3 - 5x + 1 = 0$. Take initial approximations as $x_0 = 0$, $x_1 = 0.5$, $x_2 = 1.0$. **(08 Marks)**

c. Use the Iterative method (fixed point iteration procedure) to find a real root of $\sin x = 10(x-1)$. Take $x_0 = 1.0$. **(06 Marks)**

3 a. Perform two iterations of Bairstow Method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^3 + x^2 - x + 2 = 0$. Use initial approximation as -0.9, 0.9. **(10 Marks)**

b. Find the roots of the equation $x^3 - 5x^2 - 17x + 21 = 0$ by using Graeffe's method. Carry out 3 iterations. **(10 Marks)**

4 a. For the following data, calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 24. **(10 Marks)**

| | | | | | | |
|---|---|---|---|---|---|---|
| x : | 15 | 17 | 19 | 21 | 23 | 25 |
| y : | 3.873 | 4.123 | 4.359 | 4.583 | 4.796 | 5.8 |

b. Derive Newton – Cotes formula for numerical integration and hence deduce Simpson's $\frac{1}{3}$ rd rule. Using this rule, evaluate $\int_0^1 \sqrt{1-x^2}\,dx$ by taking number of sub intervals as 8. **(10 Marks)**

5 a. Solve the system of equations $2x + y + z = 10$, $3x + 2y + 3z = 18$, $x + 4y + 9z = 16$ by the Gauss – Jordan method. **(10 Marks)**

b. Apply Cholesky method, to solve the system of equations : $x + 2y + 3z = 5$ , $2x + 8y + 22z = 6$ , $3x + 22y + 82z = -10$. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 a. Using the Jacobi method, find all the eigen values and eigen vectors of the symmetric matrix

$$A = \begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}.$$ **(10 Marks)**

b. Find all the eigen values of the matrix using the Ruti – Shauser method.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{pmatrix}$$ Carry out 3 iterations. **(10 Marks)**

7 a. Let T be a linear operator on $R^3$ defined by T (x, y, z) = (2y + z , x – 4y, 3x). Find the matrix of the transformation with respect to basis (1, 1, 1) , (1, 1, 0) , (1, 0, 0). **(10 Marks)**

b. Find a Least squares solution to AX = B with

$$A = \begin{pmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}.$$ Also compute the least squares error. **(10 Marks)**

8 a. Let V be an inner product space. Let $\{u_1, u_2, u_3, \ldots, u_n\}$ be a set of non – zero, mutually orthogonal vectors of V. Then prove that

i) the set $\{u_1, u_2, \ldots u_n\}$ is linearly independent

ii) $\left\|\sum_{i=1}^{i=n} \alpha_i u_i\right\|^2 = \sum_{i=1}^{i=n} |\alpha_i|^2 . \|u_i\|^2$. **(10 Marks)**

b. Let {(1, -1, 1, 1) , (1, 0, 1, 0) , (0, 1, 0, 1) be a linearly independent set in $R^4$. Find an Orthonormal set $\{V_1, V_2, V_3\}$ such that

L {(1, -1, 1, 1) , (1, 0, 1, 0) , (0, 1, 0, 1)} = L $(V_1, V_2, V_3)$. **(10 Marks)**

*****