

Fifth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Modern Control Theory

Time: 3 hrs.

Max. Marks:100

**Note: Answer any FIVE full questions, selecting
atleast TWO questions from each part.**

PART – A

- 1 a. Define the following terms:
 i) State ii) State variables iii) State vector iv) State space. (04 Marks)
 b. Obtain the state model for the electrical circuit shown in Fig.Q1(b). (08 Marks)

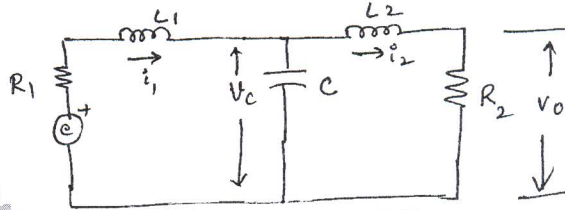


Fig.Q.1(b)

- c. Derive the state model for the system described by the differential equation
 $D^3y + 4D^2y + 5Dy + 2y = 2D^2u + 6Du + 5u$, where $D = d/dt$, in Jordan canonical form. (08 Marks)
- 2 a. Obtain the state model in first companion form for the system given by
 $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 4y = 10u$. (06 Marks)
- b. Fig.Q.2(b) shows block diagram of a control system using state variable feedback and integral control. State model of plant is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

 Derive the state model of entire system. (08 Marks)

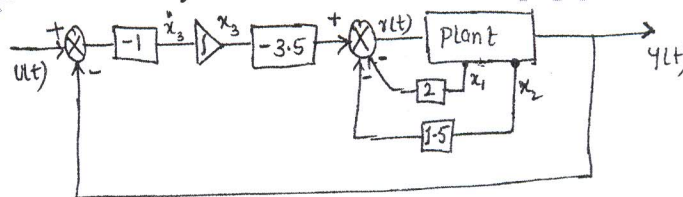


Fig.Q.2(b)

- c. List out atleast one advantage and one disadvantage of selecting,
 i) Physical variables; ii) Phase variable; iii) Canonical variables for state space formulation of control systems. (06 Marks)
- 3 a. Find eigen values, eigen vectors and modal matrix for

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 0 & 0 \\ 8 & 2 & -5 \end{bmatrix}$$
. (06 Marks)
- b. Convert the following state model into canonical form by diagonalising matrix 'A'.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
. (08 Marks)

- c. Find the state transition matrix e^{At} for the system using Cayley-Hamilton method

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

(06 Marks)

- 4 a. Find the time response for unit step input of a system given by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u(t),$$

$$y(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) \text{ and } x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(12 Marks)

- b. Evaluate controllability and observability of the following state model,

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & -3 \\ 0 & 1 & -4 \end{bmatrix} x + \begin{bmatrix} 40 \\ 10 \\ 0 \end{bmatrix} u, y = [0 \ 0 \ 1] x.$$

(08 Marks)

PART - B

- 5 a. "Any unstable system can be stabilized by complete state feedback if all the state variables are controllable". Comment on this statement and explain for the Fig.Q.5(a). (06 Marks)

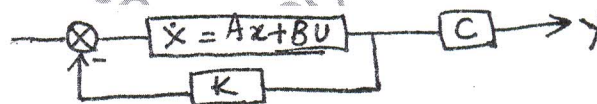


Fig.Q.5(a)

- b. Consider the system defined by $\dot{x} = Ax + Bu$, $y = cx$ where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

By using the state feedback control $u = -kx$, it is desired to have the closed loop poles at $s = -1 \pm j2$ and $s = -10$. Determine the state feedback gain matrix 'K' using Ackermann's formula. (06 Marks)

- c. A regulator system has the plant.

$$\dot{x} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, y = [0 \ 0 \ 1] x$$

Design an observer such that the eigen values of the observer are located at $-2 \pm j\sqrt{12}$ and -5 . Use direct substitution method. (06 Marks)

- 6 a. Write a short note on P, PI and PID controllers. (06 Marks)
 b. List the properties of nonlinear systems. (06 Marks)
 c. Draw the input-output characteristics of following nonlinearities and explain in detail:
 i) Dead zone ii) Backlash. (08 Marks)

- 7 a. Explain the phenomenon of jump resonance with respect to the nonlinear systems. (06 Marks)
b. Find out the singular points for the following: $\ddot{x} + 0.5\dot{x} + 2x = 0$. (04 Marks)
c. Draw the phase plane trajectory for the following equation using isocline method:

$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2x = 0. \text{ Given } \xi = 0.5, \omega_n = 1, \text{ with } x = 0 \text{ and } \frac{dx}{dt} = 1 \text{ as initial condition.}$$

(10 Marks)

- 8 a. Define the following terms:
i) Positive definiteness.
ii) Positive semidefiniteness.
iii) Negative definiteness.
iv) Negative semidefiniteness.
Give one example to each. (06 Marks)
b. Explain the Liapunov second method and stability theorems. (08 Marks)
c. Use Krasovskii's theorem to show that the equilibrium state $\mathbf{x} = 0$ of the system described by $\dot{x}_1 = -3x_1 + x_2$; $\dot{x}_2 = x_1 - x_2 - x_2^3$ is asymptotically stable in large. (06 Marks)
